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STABILITY ANALYSIS OF LARGE SPACE STRUCTURE CONTROL SYSTEMS WITH DELAYED INPUT

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Abstract

Large space structural systems, due to their inherent flexibility and low mass to area ratio, are represented by large dimensional mathematical models. For implementation of the control laws for such systems a finite amount of time is required to evaluate the control signals; and this time delay may cause instability in the closed loop control system that was previously designed without taking the input delay into consideration. The stability analysis of a simple harmonic oscillator representing the equation of a single mode as a function of delay time is analyzed analytically and verified numerically. The effect of inherent damping on the delay is also analyzed. The control problem with delayed input is also formulated in the discrete time domain.

I. Introduction

Large flexible space structures have been proposed for possible use in communications, electronic orbital based mail systems, and solar energy collection.[1,2] The size and the low mass to area ratio of such systems warrant the consideration of the flexibility as the main contribution to the dynamics and control problem as compared to the inherently rigid nature of earlier spacecraft systems. For such large flexible systems, both orientation and surface shape control may often be required.

The equations of motion describing the shape of any large space structure are either represented by a few partial differential equations or a large number of ordinary differential equations. As the partial differential equations are difficult to solve for control system design purposes, the structural dynamics are commonly described using Finite Element Methods (FEM). Two typical large space structures namely the Hoop/Column antenna[3] and the Space Station initial operational configuration (IOC)[4] are both described using 672 degrees of freedom. Thus the dynamics of a large space structure can be written as[5]:

$$M \ddot{Z} + K Z = U_C \tag{1}$$

where

$M = NXN$ mass/inertia symmetric matrix

$K = NXN$ stiffness symmetric matrix

$Z = NX1$ generalize coordinates representing the degrees of freedom

$U_C =$ influence of the external forces in each degree of freedom $= B'U$.

With the modal transformation

$$Z = \phi\, q$$

and the properties of the modal transformation such as

$$\phi^T M \phi = I$$

$$\phi^T K \phi = \text{diag}\ [\omega_1^2\ \omega_2^2\ \ldots.,\ \omega_n^2]$$

and neglecting the higher modes, equation (1) can be written in standard state space form as

$$\dot{X} = AX + BU \tag{2}$$

where

$X = 2nx1$ state vector representing modal coordinates and their velocities $[q,\dot{q}]^T$

$U = mx1$ control vector

$$A = \left[\begin{array}{c|c} 0 & I_{nxn} \\ \hline \begin{matrix} -\omega_1^2 & \\ & -\omega_n^2 \end{matrix} & 0 \end{array}\right] \quad \text{system matrix}$$

$$B = \left[\begin{array}{c} 0_{nxm} \\ \hline \phi^T B'_{nxm} \end{array}\right] \quad \text{control influence matrix}$$

II. Control with Delayed Input

The proposed control systems for large space structures are based on state variable feedback of the form:

$$U = -FX \tag{3}$$

and the control gain matrix, F, is designed using techniques such as the linear quadratic regulator (LQR) theory[6], pole placement[7], and/or linear quadratic Gaussian/loop transfer recovery (LQG/LTR).[8]

For the case when the complete state is not available for feedback, an estimate of the state, $\hat{X}$, is obtained using an appropriate estimator from the measurements of the form

$$Y = CX \tag{4}$$

where

$Y = \ell x1$ measurement vector

$C = \ell xn$ sensor influence matrix

In general, it is assumed that the estimated state, $\hat{X}$, is instantaneously available. As the state estimator is implemented using a digital computer and the number of the status (2n) is of the order of hundreds for a large space structure, the computational time becomes appreciable. Thus, in the present paper, the stability of the closed loop control system, with the control as given in equation (2), is analyzed as a function of the delay time (h) using the modified control law of the form:

$$U(t) = -FX(t-h) \tag{5}$$

The characteristic equation of the closed loop system

$$\dot{X} = AX(t) - BFX(t-h) \tag{6}$$

is given by

$$G(s,h) = \det\,(sI - A + BFe^{-sh}) = 0 \tag{7}$$

which, in turn, can be written as

$$G(s,h) = \sum_{i=0}^{2n} P_i(s)e^{-shi} = 0. \tag{8}$$

The roots of the characteristic equation, (8), as a function of the delay, h, are obtained from the corresponding auxiliary equation[9]

$$G'(s,h) = \sum_{i=0}^{2n} P_i(s)(1-Ts)^{2i}\,(1+Ts)^{4n-2i} = 0 \tag{9}$$

where

$$e^{-sh} = \left[\frac{1-sT}{1+sT}\right]^2. \tag{10}$$

The value of T for which the roots of the equation (9) cross the imaginary axis in the s-plane is obtained and the corresponding h is evaluated using the relation, (10).

III. Example of a Harmonic Oscillator

The equation of motion representing the i^{th} structural mode is the familiar harmonic oscillator and is given by

$$\ddot{x}_i + \omega_i^2 x_i = f_i \tag{11}$$

Considering the delayed velocity feedback of the form

$$f_i = -2\zeta_i\omega_i\dot{x}_i \ (t-h) \tag{12}$$

with

$$\omega_i = 6, \quad \zeta_i = 0.5,$$

the characteristic equation is given by

$$G(s,h) = s^2+36+6se^{-sh} = 0 \tag{13}$$

$$= \sum_{i=0}^{1} P_i(s)e^{-shi} = 0$$

where

$$P_o(s) = s^2 + 36$$

$$P_1(s) = 6s$$

The corresponding auxilary equation is given by

$$\sum_{i=0}^{1} P_i(s) \ (1-Ts)^{2i} \ (1+TS)^{2-2i} = 0 \tag{14}$$

i.e.

$$(s^2+36) \ (1+Ts)^2 + 6s(1-Ts)^2 = 0$$

or

$$T^2s^4 + (2T + 6T^2) \ s^3+(1+36T^2-12T)s^2$$
$$+ (72T+6)s+36 = 0 \tag{15}$$

Using the Routh-Hurwitz criterion, it can be found that the roots of equation (15) cross the imaginary axis at $\omega \doteq 9.7$ for $T \doteq 0.0426$. The corresponding delay (h) can be calculated from the relation (10) with $s = j\omega$ and is 0.16. This result can also be verified directly for this simple system with the substitution $s=j\omega$ into equation (13)[10], resulting in the value of ω and delay h for which the roots of the characteristic equation cross the imaginary axis.

Thus, equation (13) can be written as (keeping ζ_i and ω_i):

$$(\omega_i^2-\omega^2) + j(2\zeta_i\omega_i\omega)e^{-j\omega h} = 0 \tag{16}$$

or

$$(\omega_i^2-\omega^2+2\zeta_i\omega_i\omega \sin\omega h) + j2\zeta_i\omega_i\omega \cos\omega h = 0 \ . \tag{17}$$

For equation (17) to be satisfied

$$\cos \omega h = 0 \quad \text{or} \quad \omega h = \pi/2 \tag{18}$$

and

$$\omega_i^2-\omega^2+2\zeta_i\omega_i\omega = 0 \tag{19}$$

or

$$\omega = \zeta_i\omega_i \pm \omega_i\sqrt{1+\zeta_i^2}$$

Taking the positive value for ω, the delay h, is given by

$$h = \frac{\pi/2}{\omega_i[\zeta_i+\sqrt{1+\zeta_i^2}\,]} \ . \tag{20}$$

The value of h for $\zeta_i = 0.5$ and $\omega_i = 6$ is 0.16 and thus the earlier result is verified. It is observed that an increase in damping reduces the tolerable delay (h) in the input.

The equation of motion of a single mode with inherent (natural) damping and velocity feedback can be written as:

$$\ddot{X}+2\zeta_i'\omega_i\dot{X}+\omega_i^2X = f = -2\zeta_i\omega_i\dot{X}(t-h) \tag{21}$$

where ζ_i' is the inherent damping ratio.

The corresponding characteristic equation is given by

$$s^2+2\zeta_i'\omega_i s+\omega_i^2+2\zeta_i\omega_i s e^{-sh} = 0 \ . \tag{22}$$

After substituting $s = j\omega$, equation (22) can be written as:

$$(\omega_i^2-\omega^2+2\zeta_i\omega_i\omega \sin\omega h) + j(2\zeta_i'\omega_i\omega+2\zeta_i\omega_i\omega\cos\omega h) = 0 \tag{23}$$

For equation (23) to be satisfied for all ω and h, we have

$$2\zeta_i'\omega_i+2\zeta_i\omega_i \ \cos\omega h = 0 \tag{24}$$

or

$$\cos \omega h = -\zeta_i'/\zeta_i \tag{25}$$

Thus, for $\cos\omega h = < 1$, the inherent damping must be less than damping due to control for instability. For $\zeta_i < \zeta_i'$, the system will always be stable.

With the value of ωh from equation (25) the frequency ω can be calculated as:

$$\omega = \omega_i \left[\sqrt{\zeta_i^2 - \zeta_i'^2} \pm \sqrt{1+\zeta_i^2 - \zeta_i'^2}\right] \tag{26}$$

and selecting the positive value of ω, h is given by:

$$h = \frac{\cos^{-1}(-\zeta_i'/\zeta_i)}{\omega_i\left[\sqrt{\zeta_i^2 \zeta_i'^2} \pm \sqrt{1+\zeta_i^2 \zeta_i'^2}\right]} \tag{27}$$

For $\zeta_i = \zeta'$ it can be seen that the delay, h, is half the undamped natural period of vibration. As the damping due to control increases, the tolerable delay (h) decreases and is in accordance with the observation made in the case without the inherent damping. The effect of inherent damping in the system is to increase the amount of delay that the system can tolerate without become unstable as compared to the case without damping

IV. Discrete Time Domain

As the controller is implemented on a digital computer, it may be more natural to consider the delayed input problem in the discrete time domain.

The equations of motion as given by equation (2) can be written in the discrete time domain as

$$X(i+1) = A_d X(i) + B_d U(i) \tag{28}$$

where

$$A_d = e^{A\Delta}, \quad B_d = \int_0^{\Delta} e^{A(t-\Delta)}\, B\, dt$$

Δ = discretization time.

The delayed input problem can be considered in discrete time in one of the two following ways:

i) Designing the controller of the form $U = -FX(i)$ without taking into consideration the delay and then examining the effect of delay on the stability of the closed-loop control system.

The control gain matrix F is designed such that the matrix $(A_d - B_d F)$ has the eigenvalues within the unit circle. Then the delay is introduced into the control law as:

$$U(i) = -FX(i-\ell) \tag{29}$$

and

$$X(i+1) = A_d X(i) - B_d FX(i-\ell) . \tag{30}$$

The stability of equation (30) can be studied using the augmented system given by

$$\underbrace{\begin{bmatrix} X(i+1) \\ X(i) \\ \\ X(i-\ell+1) \end{bmatrix}}_{\tilde{Z}(i+1)} = \underbrace{\begin{bmatrix} A_d & 0 & 0 & 0 & -B_dF \\ I & 0 & 0 & 0 & 0 \\ & & & & \\ 0 & 0 & 0 & I & 0 \end{bmatrix}}_{\tilde{A}_d} \underbrace{\begin{bmatrix} X(i) \\ X(i-1) \\ \\ X(i-\ell) \end{bmatrix}}_{\tilde{Z}(i)} \tag{31}$$

or

$$\tilde{Z}(i+1) = \tilde{A}_d \tilde{Z}(i) \qquad (25)$$

(ii) Designing the control by taking into account the delay in the input.[6,11]

Equation (28) can be modified as :

$$X(i+1) = A_d X(i) + B_d U(i-\ell) \tag{32}$$

The control law of the form $U(i) = -F\tilde{Z}(i)$ can be designed from the augmented system:

$$\begin{bmatrix} X(i+1) \\ U(i) \\ U(i-1) \\ \\ U(i-\ell+1) \end{bmatrix} = \begin{bmatrix} A_d & 0 & 0 & 0 & B_d \\ 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ & & & & \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \underbrace{\begin{bmatrix} X(i) \\ U(i-1) \\ \\ \\ U(i-\ell) \end{bmatrix}}_{\tilde{Z}(i)} + \begin{bmatrix} 0 \\ I \\ 0 \\ \\ 0 \end{bmatrix} U(i) \tag{33}$$

or $$\tilde{Z}(i+1) = \tilde{A}_d Z(i) + \tilde{B}_d U(i) .$$

Thus the input $U(i-\ell)$ is a function of the previous inputs, $U(i-\ell-1)$, $U(i-\ell-2),\ldots,$ and the previous states $X(i-\ell)$. Though this design can take delay into consideration, the sequence of the control signals: $U(i-\ell)$, $U(i-\ell+1),\ldots$ must be generated at an interval of one step and, thus, the original delay problem is not completely solved.

Conclusions

The effect of delay in the input on the stability of the continuous time controller that is designed without taking this delay into consideration is presented. The closed-loop control system of a second order plant becomes unstable for a delay of 0.16 seconds, which is only 16 percent of its natural period of motion. It is also observed that even a small amount of inherent (natural) damping in the system can increase the amount of delay that can be tolerated without the system becoming unstable. The delay problem is formulated in the discrete time domain and an analysis procedure is suggested.

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